## EE 454 OPTICAL COMMUNICATION SYSTEMS <br> HOMEWORKS AND ANSWERS

## HOMEWORK-1

1. Show the elctromagnetic spectrum that covers the frequencies and the wavelengths involved in optical communication systems.

Answer:
The infra-red, visible and ultra-violet regions of the electromagnetic spectrum are shown below:

2. What is the distance that the optical wave whose frequency is 193.548387096774 THz travels within one cycle when it propagates in vacuum?

Answer:

$$
\begin{aligned}
\lambda(\text { in } \mathrm{m}) & =c / f=\frac{3 \times 10^{8}(\mathrm{in} \mathrm{~m} / \mathrm{sec})}{f(\text { in } 1 / \mathrm{sec}=\mathrm{Hz})}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{sec}}{193.548387096774 \mathrm{THz}}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{sec}}{193.548387096774 \times 10^{12}} \\
& =1.55 \times 10^{-6} \mathrm{~m}=1.55 \mu \mathrm{~m}
\end{aligned}
$$

3. Write the factors that exist in an optical communication system.

Answer:

- The overall cost of the system,
- Frequency of the light source (carrier frequency),
- Frequency should match the operating frequency of the detector,
- Bandwidth of the light source (causes dispersion, i.e., brodening of the received pulse),
- Modulation rate (bir rate with unit bit/second). Limited by the switching electronics. Modulation rate determines the rate of information, i.e., what amount of data can be sent in one second,
- Medium losses (attenuation). Free space loss, absorption and scattering,
- Dispersion in the medium, i.e., widening of the received pulse which results in the reduction of the bit rate that can be used,
- Detector operating frequency that should match the source carrier frequency,
- Detector response time which should be fast enough to accomodate the modulation rate

4. What equipment is used in an optical communication link if the distance is very long?

## Answer: Repeater.

5. Write 4 types of optical communication system.

Answer:
a) Free space systems. Space-to-space satellite links. (Unguided).
b) Atmospheric systems. Horizontal terrestrial links, earth to satellite links, satellite to earth links. (FSO, Free Space Optics) (Unguided)
c) Oceanic (underwater) systems. Guided systems with optical fiber and unguided wireless systems.
d) Optical fiber systems under and above the ground. Guided.
6. An analog voice signal whose maximum frequency is 4 kHz is digitized in which the sampled signal is quantized with 256 levels. What is the data rate of the digitized voice signal?

Answer: Using Nyquist sampling rate, sampling frequency is $4 \mathrm{kHz} \mathrm{x} 2=8 \mathrm{kHz}$, i.e., $8000 \mathrm{samples} / \mathrm{sec}$
256 levels of quantization means that a sample is presented by 8 bits, i.e., 8 bits / sample
Thus, the data rate of the digitized voice signal is ., 8000 samples / sec x 8 bits / sample $=64 \mathrm{kbits} / \mathrm{sec}$

## HOMEWORK-2

7. Draw the block diagram of a basic optical communication system architecture.

Answer:

8. What are the main components of optical fiber systems?

Answer:

- The optical source: LED (semiconductor), laser diode (semiconductor),
- A means of modulating the optical output from the source with the signal to be transmitted (internal modulation),
- The transmission medium: Step index fibers (single mode, multimode), graded index fibers (multimode),
- The photodetector which converts the received the received optical power back into an electrical signal. Pin (semiconductor), avalanche (semiconductor),
- Electronic amplification and signal processing required to recover the signal and present it in a form suitable to use,
- Connectors (source to fiber, fiber to fiber, fiber to photo detector) and splicing (fiber to fiber).

9. a. If the refractive indices of medium 1 and 2 are $n_{1}=1.54$ and $n_{2}=1.50$, respectively, find the critical angle.
b. At which angles, total internal reflection occurs and the light ray is totally reflected at the interface, thus stays totally in the medium 1 ?
c. If medium 1 is the core and medium 2 is the cladding, find the numerical aperture ( $N A$ ) which is a measure of light gathering power (acceptance cone) of the optical fiber when the refractive index of air is 1 .
d. At which angles of incidence to fiber, the fiber guides the light without refraction occurring at the core-cladding interface?

Answer: a.
$\theta_{c}=\arcsin \left(n_{2} / n_{1}\right)=\arcsin (1.50 / 1.54)=\arcsin (0.974025974025974)=1.34237893242878 \mathrm{rad}$ $=180 \times 1.34237893242878 / \pi=76.9149157527231^{\circ}$
b. At angles $>76.9149157527231^{\circ}$, total internal reflection occurs and the light ray is totally reflected at the interface, thus stays totally in the medium 1.
c. $n_{a} \sin \alpha_{m}$ is defined as the numerical aperture (NA) which is a measure of light gathering power (acceptance cone) of the optical fiber. For $n_{a}=1$, the numerical aperture is $N A=\sin \alpha_{m}=\left(n_{1}^{2}-n_{2}^{2}\right)^{0.5}=\left[(1.54)^{2}-(1.50)^{2}\right]^{0.5}=(2.3716-2.25)^{0.5}=0.348712$.
d. In order for the fiber to guide the light without refraction occurring at the core-cladding interface, angle of incidence to fiber $(\alpha)$ should be less than $\alpha_{m}$ where $\sin \alpha_{m}=0.348712$,
i.e., $\alpha_{m}=\arcsin (0.348712)=0.356196 \mathrm{rad}=180 \times 0.316196 / \pi=20.40915^{\circ}$

Thus, for $\alpha<20.40915^{\circ}$, the fiber guides the light without refraction occurring at the core-cladding interface.
10. What are the 2 methods used to formulate the propagation of light in the optical fiber?

Answer: Wave optics and ray optics.

## HOMEWORK-3

11. Assume that the propagation is in $z$-direction with a longitudinal propagation constant $\beta$, i.e., $\beta$ is the longitudinal component of the propagation vector $\beta$. Assume that the permittivity $\varepsilon(x, y)$ does
not depend on $z$ but can vary with $x$ and $y$. However, the permittivity is assumed to vary in very small amounts over the region of the wavelength so is assumed to be constant. Write the electric field and the magnetic field in a waveguide showing the components of these fiels for $e^{j o t}$ time dependence.

Answer:
$\overline{\mathrm{E}}=E_{x} \hat{\mathrm{a}}_{x}+E_{y} \hat{\mathrm{a}}_{y}+E_{z} \hat{\mathrm{a}}_{z}$ and $\overline{\mathrm{H}}=H_{x} \hat{\mathrm{a}}_{x}+H_{y} \hat{\mathrm{a}}_{y}+H_{z} \hat{\mathrm{a}}_{z}$

For $e^{j \omega t}$ time dependence $E_{x}=E_{x 0}(x, y) e^{-j \beta z} e^{j \omega t}, E_{y}=E_{y 0}(x, y) e^{-j \beta z} e^{j \omega t}, E_{z}=E_{z 0}(x, y) e^{-j \beta z} e^{j \omega t}$
12. Write the Maxwell's equations as used in finding the modes in the fiber.

Answer:
$\nabla \times \overline{\mathrm{E}}=-\frac{\partial \overline{\mathrm{B}}}{\partial t}=-\mu \frac{\partial \overline{\mathrm{H}}}{\partial t}$ and $\nabla \times \overline{\mathrm{H}}=\frac{\partial \overline{\mathrm{D}}}{\partial t}=\varepsilon \frac{\partial \overline{\mathrm{E}}}{\partial t}$
13. Using the Maxwell's equations, derive the transverse components of the electric and the magnetic fields (i.e., $E_{x}, E_{y}, H_{x}, H_{y}$ ) in terms of their longitudinal components (i.e., $E_{z}, H_{z}$ ).

Answer: Using the Maxwell's equations we have

$$
\begin{aligned}
& \left(\frac{\partial E_{z}}{\partial y}-\frac{\partial E_{y}}{\partial z}\right) \hat{\mathbf{a}}_{x}+\left(\frac{\partial E_{x}}{\partial z}-\frac{\partial E_{z}}{\partial x}\right) \hat{\mathbf{a}}_{y}+\left(\frac{\partial E_{y}}{\partial x}-\frac{\partial E_{x}}{\partial y}\right) \hat{\mathbf{a}}_{z}=-\mu \frac{\partial H_{x}}{\partial t} \hat{\mathbf{a}}_{x}-\mu \frac{\partial H_{y}}{\partial t} \hat{\mathbf{a}}_{y}-\mu \frac{\partial H_{z}}{\partial t} \hat{\mathbf{a}}_{z} \\
& \left(\frac{\partial H_{z}}{\partial y}-\frac{\partial H_{y}}{\partial z}\right) \hat{\mathbf{a}}_{x}+\left(\frac{\partial H_{x}}{\partial z}-\frac{\partial H_{z}}{\partial x}\right) \hat{\mathbf{a}}_{y}+\left(\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}\right) \hat{\mathbf{a}}_{z}=\varepsilon \frac{\partial E_{x}}{\partial t} \hat{\mathbf{a}}_{x}+\varepsilon \frac{\partial E_{y}}{\partial t} \hat{\mathbf{a}}_{y}+\varepsilon \frac{\partial E_{z}}{\partial t} \hat{\mathbf{a}}_{z}
\end{aligned}
$$

Using the answer in question $11, \frac{\partial E_{x}}{\partial t}=j \omega E_{x}$ and $\frac{\partial E_{x}}{\partial z}=-j \beta E_{x}$

$$
\begin{aligned}
& \frac{\partial H_{z}}{\partial y}+j \beta H_{y}=j \omega \varepsilon E_{x},-j \beta H_{x}-\frac{\partial H_{z}}{\partial x}=j \omega \varepsilon E_{y}, \frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}=j \omega \varepsilon E_{z} \\
& \frac{\partial E_{z}}{\partial y}+j \beta E_{y}=-j \omega \mu H_{x},-j \beta E_{x}-\frac{\partial E_{z}}{\partial x}=-j \omega \mu H_{y}, \frac{\partial E_{y}}{\partial x}-\frac{\partial E_{x}}{\partial y}=-j \omega \mu H_{z}
\end{aligned}
$$

To express $E_{x}, E_{y}, H_{x}, H_{y}$ in terms of $E_{z}, H_{z}$, we use $\frac{\partial H_{z}}{\partial y}+j \beta H_{y}=j \omega \varepsilon E_{x}$ and $-j \beta E_{x}-\frac{\partial E_{z}}{\partial x}=-j \omega \mu H_{y}$ to obtain $j \omega \varepsilon E_{x}=\frac{\partial H_{z}}{\partial y}+\frac{j \beta}{-j \omega \mu}\left(-j \beta E_{x}-\frac{\partial E_{z}}{\partial x}\right)$. Multiplying by $-j \omega \mu$ and rearranging $E_{x}=-\frac{j}{\kappa^{2}}\left(\omega \mu \frac{\partial H_{z}}{\partial y}+\beta \frac{\partial E_{z}}{\partial x}\right)$ where $\kappa^{2}=k^{2}-\beta^{2}$ and $k^{2}=\omega^{2} \mu \varepsilon$

Using $-j \beta H_{x}-\frac{\partial H_{z}}{\partial x}=j \omega \varepsilon E_{y}$ and $\frac{\partial E_{z}}{\partial y}+j \beta E_{y}=-j \omega \mu H_{x}$, we obtain $E_{y}=-\frac{j}{\kappa^{2}}\left(\beta \frac{\partial E_{z}}{\partial y}-\omega \mu \frac{\partial H_{z}}{\partial x}\right)$
From $\frac{\partial E_{z}}{\partial y}+j \beta E_{y}=-j \omega \mu H_{x}$ and $-j \beta H_{x}-\frac{\partial H_{z}}{\partial x}=j \omega \varepsilon E_{y}$, we obtain $H_{x}=-\frac{j}{\kappa^{2}}\left(\beta \frac{\partial H_{z}}{\partial x}-\omega \varepsilon \frac{\partial E_{z}}{\partial y}\right)$
From $-j \beta E_{x}-\frac{\partial E_{z}}{\partial x}=-j \omega \mu H_{y}$ and $\frac{\partial H_{z}}{\partial y}+j \beta H_{y}=j \omega \varepsilon E_{x}$, we obtain $H_{y}=-\frac{j}{\kappa^{2}}\left(\beta \frac{\partial H_{z}}{\partial y}+\omega \varepsilon \frac{\partial E_{z}}{\partial x}\right)$
14. Using the equations found in question 13 , find the wave equations in $E_{z}$ and $H_{z}$.

Answer: Substituting the last two equations in the answers to question 13 into $\frac{\partial E_{z}}{\partial y}+j \beta E_{y}=-j \omega \mu H_{x}$ and multiplying by $\frac{j \kappa^{2}}{\omega \varepsilon}$, we have $\nabla_{\mathrm{T}}^{2} E_{z}+\kappa^{2} E_{z}=0$ where $\nabla_{\mathrm{T}}^{2}=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}$.

Similarly for $H_{z}, \nabla_{\mathrm{T}}^{2} H_{z}+\kappa^{2} H_{z}=0$ is found.

## HOMEWORK-4

15. What longitudinal and transverse components of TEM (Transverse Electromagnetic), TE (Transverse Electric), TM (Transverse Magnetic), HE and EH (Hybrid ) modes.

Answer:

| Mode | Longitudinal <br> Components | Transverse <br> Components |
| :--- | :---: | :---: |
| TEM | $E_{z}=0, H_{z}=0$ | $E_{x}, E_{y}, H_{x}, H_{y}$ |
| TE | $E_{z}=0, H_{z} \neq 0$ | $E_{x}, E_{y}, H_{x}, H_{y}$ |
| TM | $E_{z} \neq 0, H_{z}=0$ | $E_{x}, E_{y}, H_{x}, H_{y}$ |
| HE or EH | $E_{z} \neq 0, H_{z} \neq 0$ | $E_{x}, E_{y}, H_{x}, H_{y}$ |

16. In cylindrical coordinates, write the transverse components of the electric and the magnetic fields (i.e., $E_{r}, E_{\phi}, H_{r}, H_{\phi}$ ) in terms of their longitudinal components (i.e., $E_{z}, H_{z}$ ).

Answer: By transforming $\frac{\partial^{2} E_{z}}{\partial r^{2}}+\frac{1}{r} \frac{\partial E_{z}}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} E_{z}}{\partial \phi^{2}}+\kappa^{2} E_{z}=0$ and $\frac{\partial^{2} H_{z}}{\partial r^{2}}+\frac{1}{r} \frac{\partial H_{z}}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} H_{z}}{\partial \phi^{2}}+\kappa^{2} H_{z}=0$

$$
\begin{aligned}
& E_{r}=-\frac{j}{\kappa^{2}}\left(\beta \frac{\partial E_{z}}{\partial r}+\omega \mu \frac{1}{r} \frac{\partial H_{z}}{\partial \phi}\right), E_{\phi}=-\frac{j}{\kappa^{2}}\left(\beta \frac{1}{r} \frac{\partial E_{z}}{\partial \phi}-\omega \mu \frac{\partial H_{z}}{\partial r}\right), \\
& H_{r}=-\frac{j}{\kappa^{2}}\left(\beta \frac{\partial H_{z}}{\partial r}+\omega \varepsilon \frac{1}{r} \frac{\partial E_{z}}{\partial \phi}\right), H_{\phi}=-\frac{j}{\kappa^{2}}\left(\beta \frac{1}{r} \frac{\partial H_{z}}{\partial \phi}-\omega \varepsilon \frac{\partial E_{z}}{\partial r}\right)
\end{aligned}
$$

17. a. What does ray optics describe?
b. Starting with the Helmholtz equation, derive the eikonal equation.
c. What does the eikonal equation determine?
d. How are the light rays defined in ray optics?

Answer:
a. Ray optics describes the propagation of light in the form of rays.
b. Starting with the Helmholtz equation $\nabla^{2} \overline{\mathrm{E}}+k^{2} \overline{\mathrm{E}}=0$

If $\Psi$ is any rectangular component of $\overline{\mathrm{E}}$, then $\nabla^{2} \Psi+k^{2} \Psi=0$ where $k=n k_{0}=n\left(\frac{2 \pi}{\lambda_{0}}\right)$,
$k_{0}$ being the propagation constant in vacuum.
The solution is in the form $\Psi=\Psi_{o}(x, y, z) \mathrm{e}^{-j k_{0} S(x, y, z)}$ where $\Psi_{o}$ and $S$ are real functions of position. $S(x, y, z)$ is the phase function associated with the medium and is called the "eikonal".

Substituting $\Psi=\Psi_{o}(x, y, z) \mathrm{e}^{-j k_{0} S(x, y, z)}$ in $\nabla^{2} \Psi+k^{2} \Psi=0$, after some algebra it can be shown that for $\lambda_{0} \rightarrow 0, \nabla S=n$, known as the eikonal equation.
c. The eikonal equation determines the function $S$ that defines the surfaces of constant phase by the equation $S(x, y, z)=$ constant.
d. The light rays are defined as the locus of points that form the orthogonal trajectories to the constant phase fronts of a light wave, i.e., if constant phase surfaces are known, we can construct the light rays by drawing lines perpendicular to the phase fronts. However, it is often desirable to find the ray trajectories directly without having to construct the phase fronts.
18. a. Without derivation, write the paraxial ray equations in cylindrical coordinates $(r, \phi, z)$.
b. Draw the rays in a fiber core showing the ray paths of the highest order and the lowest order modes.

Answer: a. $\frac{\partial^{2} r}{\partial z^{2}}-r\left(\frac{\partial \phi}{\partial z}\right)^{2}=\frac{1}{n_{a}} \frac{\partial n}{\partial r}$,

$$
\frac{\partial}{\partial z}\left(r^{2} \frac{\partial \phi}{\partial z}\right)=\frac{1}{n_{a}} \frac{\partial n}{\partial \phi}
$$

b.

19. a. Sketch the step index fiber showing the refractive index profile.
b. Find the solution for $E_{z}$ and $H_{z}$ in a step index fiber in cylindrical coordinates.
c. Find $E_{r}, E_{\phi}, H_{r}, H_{\phi}$ in the step index fiber.

Answer: a.

b. Equations satisfying $E_{z}$ and $H_{z}$ are

$$
\frac{\partial^{2} E_{z}}{\partial r^{2}}+\frac{1}{r} \frac{\partial E_{z}}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} E_{z}}{\partial \phi^{2}}+\kappa^{2} E_{z}=0 \text { and } \frac{\partial^{2} H_{z}}{\partial r^{2}}+\frac{1}{r} \frac{\partial H_{z}}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} H_{z}}{\partial \phi^{2}}+\kappa^{2} H_{z}=0
$$

Longitudinal direction of propagation is in $z$-direction, time dependence is $j \omega t$ so that fields have dependence of the form $\mathrm{e}^{j(\omega t-\beta z)}$

Using separation of variables
$E_{z}(\phi, r)=A \Phi(\phi) F(r)$
and since fiber has circular symmetry, we choose
$\Phi(\phi)=\mathrm{e}^{j v \phi}$ where $v$ is $(+)^{\prime}$ we or $(-)$ 'se integer and
$E_{z}(\phi, r)=A F(r) \mathrm{e}^{j v \phi}$.

Using this in $\frac{\partial^{2} E_{z}}{\partial r^{2}}+\frac{1}{r} \frac{\partial E_{z}}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} E_{z}}{\partial \phi^{2}}+\kappa^{2} E_{z}=0$ and multiplying by $1 / \mathrm{e}^{j \nu \phi}$, it is found that $\frac{d^{2} F(r)}{d r^{2}}+\frac{1}{r} \frac{d F(r)}{d r}+\left(\kappa^{2}-\frac{v^{2}}{r^{2}}\right) F(r)=0$

This is the form of Bessel's equation. The solution should have the following properties:
i) The field in the core must be finite at $r=0$.
ii) Cladding field must have an exponentially decaying behaviour at large distances from the center of the fiber.

The proper solution for $E_{z}(\phi, r)=A \Phi(\phi) F(r)$ is
$E_{z}=\left\{\begin{array}{ll}A J_{v}(\kappa r) \mathrm{e}^{j v \phi}, & r<\mathrm{a} \\ C H_{v}^{(1)}(j \gamma r) \mathrm{e}^{j v \phi}, & r>\mathrm{a}\end{array}\right.$ and similarly $H_{z}= \begin{cases}B J_{v}(\kappa r) \mathrm{e}^{j v \phi}, & r<\mathrm{a} \\ D H_{v}^{(1)}(j \gamma r) \mathrm{e}^{j v \phi}, & r>\mathrm{a}\end{cases}$
where $J_{v}$ is the Bessel function of order $v$ and $H_{v}^{(1)}$ is the modified Hankel function of the first kind of order $v$.
$A, B, C, D$ are unknown constants.
c. From Maxwell's equations the transverse fields in the core, i.e., $E_{r}, E_{\phi}, H_{r}, H_{\phi}$ for $r<\mathrm{a}$ are
$E_{r}=-\frac{j}{\kappa^{2}}\left[A \beta \kappa J_{v}^{\prime}(\kappa r)+B(j v)(\omega \mu) \frac{1}{r} J_{v}(\kappa r)\right] \mathrm{e}^{j \nu \phi}, \quad E_{\phi}=-\frac{j}{\kappa^{2}}\left[j \beta \frac{v}{r} A J_{v}(\kappa r)-\kappa \omega \mu B J_{v}^{\prime}(\kappa r)\right] \mathrm{e}^{j v \phi}$,
$H_{r}=-\frac{j}{\kappa^{2}}\left[-j \omega \varepsilon_{1} \frac{v}{r} A J_{v}(\kappa r)+\kappa \beta B J_{v}^{\prime}(\kappa r)\right] \mathrm{e}^{j v \phi}, H_{\phi}=-\frac{j}{\kappa^{2}}\left[\omega \varepsilon_{1} A J_{v}^{\prime}(\kappa r)+j \beta \frac{v}{r} B J_{v}(\kappa r)\right] \mathrm{e}^{j v \phi}$
where $J_{v}^{\prime}(\kappa r)=\frac{\partial J_{v}(\kappa r)}{\partial(\kappa r)}$ and $\kappa^{2}=k_{1}^{2}-\beta^{2}=\omega^{2} \mu_{0} \varepsilon_{1}-\beta^{2}$
Similarly, the transverse field in the cladding (i.e., for $r>a$ ) are found as $E_{r}=-\frac{1}{\gamma^{2}}\left[\beta \gamma C H_{v}^{(1)}(j \gamma r)+\omega \mu_{0} \frac{v}{r} D H_{v}^{(1)}(j \gamma r)\right] \mathrm{e}^{j \nu \phi}, E_{\phi}=-\frac{1}{\gamma^{2}}\left[\beta \frac{v}{r} C H_{v}^{(1)}(j \gamma r)-\gamma \omega \mu_{0} D H_{v}^{(1)^{\prime}}(j \gamma r)\right] \mathrm{e}^{j v \phi}$,
$H_{r}=-\frac{1}{\gamma^{2}}\left[-\omega \varepsilon_{2} \frac{v}{r} C H_{v}^{(1)}(j \gamma r)-\gamma \beta D H_{v}^{(1)}(j \gamma r)\right] \mathrm{e}^{j \nu \phi}, H_{\phi}=-\frac{1}{\gamma^{2}}\left[\beta \gamma \varepsilon_{2} C H_{v}^{(1)^{\prime}}(j \gamma r)+\beta \frac{v}{r} D H_{v}^{(1)}(j \gamma r)\right] \mathrm{e}^{j \nu \phi}$
where $H_{v}^{(1)}(j \gamma r)=\frac{\partial H_{v}^{(1)}(j \gamma r)}{\partial(j \gamma r)}$ and $\gamma^{2}=\beta^{2}-k_{2}^{2}=\beta^{2}-\omega^{2} \mu_{0} \varepsilon_{2}$
$A, B, C, D$ and $\beta$ are found to be determined by applying the boundary conditions at the core-cladding interface ( $r=\mathrm{a}$ ). Boundary conditions at $r=\mathrm{a}$ are that the tangential $E$ fields and the tangential $H$ fields are equal at $r=\mathrm{a}$, i.e., $E_{z 1}=E_{z 2}$ at $r=\mathrm{a}, E_{\phi 1}=E_{\phi 2}$ at $r=\mathrm{a}, H_{z 1}=H_{z 2}$ at $r=\mathrm{a}, H_{\phi 1}=H_{\phi 2}$ at $r=\mathrm{a}$ where 1 and 2 refer to the core and the cladding, respectively.

Using the above field expressions for the core and the cladding in the boundary conditions, we obtain
$[W]\left[\begin{array}{l}A \\ B \\ C \\ D\end{array}\right]=[0]$ from which $A, B, C, D$ can be found where
$[W]=\left[\begin{array}{cccc}J_{v}(\kappa \mathrm{a}) & 0 & -H_{v}^{(1)}(j \gamma \mathrm{a}) & 0 \\ \frac{v}{\mathrm{a}} \frac{\beta}{\kappa^{2}} J_{v}(\kappa \mathrm{a}) & \frac{j \omega \mu_{0}}{\kappa} J_{v}^{\prime}(\kappa \mathrm{a}) & \frac{v}{\mathrm{a}} \frac{\beta}{\gamma^{2}} H_{v}^{(1)}(j \gamma \mathrm{a}) & -\frac{\omega \mu_{0}}{\gamma} H_{v}^{(1)}(j \gamma \mathrm{a}) \\ 0 & J_{v}(\kappa \mathrm{a}) & 0 & -H_{v}^{(1)}(j \gamma \mathrm{a}) \\ -\frac{j \omega \varepsilon_{1}}{\kappa} J_{v}^{\prime}(\kappa \mathrm{a}) & \frac{v}{\mathrm{a}} \frac{\beta}{\kappa^{2}} J_{v}(\kappa \mathrm{a}) & \frac{\omega \varepsilon_{2}}{\gamma} H_{v}^{(1)^{\prime}}(j \gamma \mathrm{a}) & \frac{v}{\mathrm{a}} \frac{\beta}{\gamma^{2}} H_{v}^{(1)}(j \gamma \mathrm{a})\end{array}\right]$
Non trivial solution gives determinant of $[W]=|W|=0$ which gives the "eigenvalue" or the characteristic equation of the waveguide. Characteristic equation is found as
$\left[\frac{\varepsilon_{1}}{\varepsilon_{2}} \frac{\mathrm{a} \gamma^{2}}{\kappa} \frac{J_{v}^{\prime}(\kappa \mathrm{a})}{J_{v}(\kappa \mathrm{a})}+j \gamma \mathrm{a} \frac{H_{v}^{(1)^{\prime}}(j \gamma \mathrm{a})}{H_{v}^{(1)}(j \gamma \mathrm{a})}\right]\left[\frac{\mathrm{a} \gamma^{2}}{\kappa} \frac{J_{v}^{\prime}(\kappa \mathrm{a})}{J_{v}(\kappa \mathrm{a})}+j \gamma \mathrm{a} \frac{H_{v}^{(1)^{\prime}}(j \gamma \mathrm{a})}{H_{v}^{(1)}(j \gamma \mathrm{a})}\right]=\left[v\left(\frac{\varepsilon_{1}}{\varepsilon_{2}}-1\right) \frac{\beta k_{2}}{\kappa^{2}}\right]^{2}$
From this equation, the characteristic equation can be found for the
$\left.\begin{array}{l}\text { TE modes when } v=0, E_{z}=0 \\ \text { TM modes when } v=0, H_{z}=0\end{array}\right\}$ meridional rays (crosses the axis)
$\left.\begin{array}{l}\mathrm{HE}\left(H_{z} \text { is the determining field }\right) \\ \mathrm{EH}\left(E_{z} \text { is the determining field }\right)\end{array}\right\}$ skew rays $(v \neq 0)$

## HOMEWORK-5

20. a. How are the cut-off frequencies obtained for the $\mathrm{TE}_{0 \mu}, \mathrm{TM}_{0 \mu}$ and $\mathrm{HE}_{v \mu}$ modes?
b. Which mode exists for all the frequencies?
c. Which mode exists in the single mode fiber?
d. Drawing the Bessel function solution, show the cut-off frequencies of some of the modes in part a.

Answer: a. Cut-off frequencies of $\mathrm{TE}_{0 \mu}, \mathrm{TM}_{0 \mu}$ modes are obtained from the $\mu^{\text {th }}$ root of $J_{0}(\kappa \mathrm{a})=0$ where $\kappa^{2}=\omega_{c}^{2} \mu_{0} \varepsilon_{1}-\omega_{c}^{2} \mu_{0} \varepsilon_{2}, \varepsilon_{1}$ is the permittivity of the core, $\varepsilon_{2}$ is the permittivity of the cladding, $\mu_{0}$ is the permeability and a is the core radius.

Cut-off frequencies of $\mathrm{HE}_{v \mu}$ modes are obtained from the $\mu^{\text {th }}$ root of $J_{v}(\kappa \mathrm{a})=0$ where $\kappa \mathrm{a}=x_{v \mu}$ for $\mu=1,2,3, x_{v \mu}$ being the $\mu^{\text {th }}$ root of $J_{v}\left(x_{v \mu}\right)=0$.
b. $\mathrm{HE}_{11}$.
c. $\mathrm{HE}_{11}$.
d.

21. The operating wavelength is $1.55 \mu \mathrm{~m}$, the refractive indices of the core, $n_{1}$ and the cladding, $n_{2}$ are 1.51239 and 1.51 , respectively.
a. Find the number of modes existing in the core if the core diameter is $50 \mu \mathrm{~m}$.
b. Find the number of modes existing in the core if the core diameter is $9 \mu \mathrm{~m}$.

Answer:
a. V number of the fiber is $V=\frac{2 \pi}{\lambda_{0}} \mathrm{a} \sqrt{n_{1}^{2}-n_{2}^{2}}=\frac{2 \pi}{1.55 \times 10^{-6}}(50 / 2) \times 10^{-6} \sqrt{(1.51239)^{2}-(1.51)^{2}} \approx 8.61$

$$
N=\frac{4 V^{2}}{\pi^{2}}=\frac{4 \times(8.61)^{2}}{\pi^{2}} \approx 30
$$

b. V number of the fiber is $V=\frac{2 \pi}{\lambda_{0}} \mathrm{a} \sqrt{n_{1}^{2}-n_{2}^{2}}=\frac{2 \pi}{1.55 \times 10^{-6}}(9 / 2) \times 10^{-6} \sqrt{(1.51239)^{2}-(1.51)^{2}} \approx 1.55$

$$
N=\frac{4 V^{2}}{\pi^{2}}=\frac{4 \times(1.55)^{2}}{\pi^{2}} \approx 1
$$

22. Which linearly polarized mode (LP) corresponds to the mode present in the single mode fiber?

Answer: $\mathrm{LP}_{01}$ corresponds to $\mathrm{HE}_{11}$.
23. Write the general formulas indicating the refractive index profiles of various graded index fibers such as the parabolic profile and draw these profiles.

Answer:


$$
\begin{aligned}
& \text { Refractive index profile } \\
& n(r)= \begin{cases}n_{1}\left[1-2 \Delta\left(\frac{r}{a}\right)^{\alpha}\right]^{1 / 2} & , r<a \\
n_{1}(1-2 \Delta)^{1 / 2} & , r>a\end{cases}
\end{aligned}
$$

$$
\underbrace{\substack{n(r)}}_{a} \rightarrow \underset{\text { radivs, }}{ } \rightarrow \begin{gathered}
\alpha=\infty \\
\text { step index } \\
\text { profile }
\end{gathered}
$$

$$
\text { where } \quad \Delta=\frac{n_{1}^{2}-n_{2}^{2}}{2 n_{1}^{2}}
$$

24. For a graded index fiber,
a. without derivation, write the paraxial ray equations from which the ray trajectories can be found.
b. find the solution of the ray trajectory, i.e., $r(z)$ and $\phi(z)$ for the general ray travelling along a spiraling trajectory.
c. find the special case solutions of meridional and helical rays from the solution of the ray trajectory, i.e., $r(z)$ and $\phi(z)$ for the general ray.
d. with parabolic profile, find the total number modes if the corresponding step index fiber has 100 modes.

Answer: a. $\frac{d}{d z}\left(r^{2} \frac{d \phi}{d z}\right)=\frac{1}{n_{1}} \frac{\partial n}{\partial \phi}, \frac{d^{2} r}{d z^{2}}-r\left(\frac{d \phi}{d z}\right)^{2}=\frac{1}{n_{1}} \frac{\partial n}{\partial r}$.
b. The refractive index does not vary with respect to $\phi$. Thus $\frac{\partial n}{\partial \phi}=0$. Using this in
$\frac{d}{d z}\left(r^{2} \frac{d \phi}{d z}\right)=\frac{1}{n_{1}} \frac{\partial n}{\partial \phi}$ and integrating, $r^{2} \frac{d \phi}{d z}=$ constant $\Rightarrow \frac{d \phi}{d z}=\frac{c_{1}}{r^{2}}$.
For $\Delta=\frac{n_{1}^{2}-n_{2}^{2}}{2 n_{1}^{2}} \ll 1, n(r)=n_{1}\left[1-\left(\frac{r}{\mathrm{a}}\right)^{2} \Delta\right]$ and $\frac{\partial n}{\partial r}=-2 n_{1}\left(\frac{\Delta}{\mathrm{a}^{2}}\right) r$
Substituting $\frac{\partial n}{\partial r}=-2 n_{1}\left(\frac{\Delta}{\mathrm{a}^{2}}\right) r$ and $\frac{d \phi}{d z}=\frac{c_{1}}{r^{2}}$ into $\frac{d^{2} r}{d z^{2}}-r\left(\frac{d \phi}{d z}\right)^{2}=\frac{1}{n_{1}} \frac{\partial n}{\partial r}$, we obtain
$\frac{d^{2} r}{d z^{2}}+2 \frac{\Delta}{\mathrm{a}^{2}} r-\frac{c_{1}^{2}}{r^{3}}=0$ which will yield (without showing the intermediate steps)
$r(z)=A\left\{1+\sqrt{1-b^{2}} \sin \left[2 \Omega\left(z-z_{0}\right)\right]\right\}^{1 / 2}$
where $A=\frac{\sqrt{c_{3}}}{\Omega}, \Omega=\frac{\sqrt{2 \Delta}}{\mathrm{a}}, b^{2}=c_{1}^{2}\left(\frac{\Omega}{c_{3}}\right), c_{3}$ and $z_{0}$ being constants.
Using the above found $r(z)$ in $\frac{d \phi}{d z}=\frac{c_{1}}{r^{2}}$ and integrating we find
$\phi(z)=\phi_{0}+\arctan \frac{1}{b}\left\{\sqrt{1-b^{2}}+\tan \left[\Omega\left(z-z_{0}\right)\right]\right\}$
where $\phi_{0}$ is a constant.
$r(z)$ and $r(z)$ describe the ray trajectory.
c. For meridional rays $\phi(z)$ does not change with respect to $z \Rightarrow c_{1}=0 \Rightarrow b=0$, it can be shown that $r(z)=\sqrt{2} A \sin \left[\Omega\left(z-z_{0}\right)+\frac{\pi}{4}\right]$.

Also for $b=0, \phi(z)=\phi_{0}+\frac{\pi}{2}$, i.e., $\phi$ is a constant. The solution for the meridional ray shows that the ray moves on a sinusoidal trajectory in the meridional plane (i.e., passing through the axis) crossing the axis $r(z)=0$ (meridional ray) as it propagates along the $z$-axis. Spatial radian frequency of this sinusoidal meridional ray is $\Omega$.

The other special case occurs when $b=1$.
$r(z)=A, \phi(z)=\phi_{0}+\left(z-z_{0}\right) \Omega$.
This ray is known as the helical ray since the ray travels at a fixed distance from the axis
$(r=A)$ on a helical path described by $\phi(z)=\phi_{0}+\left(z-z_{0}\right) \Omega$.
d. Total number of modes in a graded index fiber with a parabolic profile $(\alpha=2)$ is half the number of modes in a step index index fiber. Thus $N=100 / 2=50$.

## HOMEWORK-6

25. a. Write the 3 types of attenuation that causes power loss in optical fibers.
b. For each of the 3 types of losses given in part a, write the different mechanisms that cause these losses.

Answer:
a. Absorption losses, scattering losses and radiative losses.
b. Light is absorbed in the fiber by 3 different mechanisms:

1. By atomic defects in the glass composition. Atomic defects are imperfections of the atomic structure of the fiber such as missing molecules, high density clusters of atom groups or oxygen defects in the glass structure. Attenuation due to atomic effects is not significant in general. However, it could be important if the fiber is exposed to intense nuclear radiation levels.
2. Extrinsic absorption by impurity atoms in the glass material which is the dominant absorption factor in fibers that are prepared by direct melt method. Impurity absorption results from ions such as iron, chromium, cobalt, copper and OH (hydroxyl) ions.

Metal impurities present in the fiber is around 1 or 10 parts in $10^{9}$ which changes depending on the fiberfabrication method.

OH ion impurities in the fiber preforms result from the oxyhydrogen flme used for the hydrolysis reaction of the $\mathrm{SiCl}_{4}, \mathrm{GeCl}_{4}$ and $\mathrm{POCl}_{3}$.

OH concentration should be less than 1 in $10^{9}-10^{10}$ parts for attenuation to be at reasonable levels.
3. Intrinsic absorption by the basic constituent atoms of the fiber material.

Scattering results from microscopic variations in the material density, from compositional fluctuations or from structural inhomogeneities or defects occurring during manufacturing of the fiber.

Glass, being being composed of randomly connected molecules, contains regions in which the molecular density is either higher or lower than the average density in glass.

Also glass has several oxides such as $\mathrm{SiO}_{2}, \mathrm{GeO}_{2} \mathrm{P}_{2} \mathrm{O}_{5}$ so compositional fluctuations occur.
As a result of these two effects $n$ varies within glass over distances that are small compared to the wavelength. Result is Rayleigh scattering which has $\lambda^{-4}$, i.e., $f^{4}$ dependence. At $\lambda<1 \mu \mathrm{~m}$, Rayleigh scattering is the dominant loss in fibers.

Radiative losses: Bending losses (macrobending) are the losses due to bends in the fiber having radii that are large compared to the fiber diameter. As the redius of curvature decreases loss increases exponentially. Microbending losses are due to the microbands which are known as the repetitive changes in the radius of curvature of the fiber axis. Microbends are caused by either nonuniformities in the sheathing of the fiber or by nonuniform lateral pressures created during the cabling of the fiber (also known as the cabling or packaging losses. Microbending loss is minimized by extruding a compressible jacket over the fiber.

Waveguide losses originate from the small variations in the core diameter, which can easily arise during manufacture, may give rise to scattering.
26. a. What is dispersion?
b. What happens as the result of dispersion?
c. Find the phase velocity of the optical wave in the core if the wavelength is $1.55 \mu \mathrm{~m}$ and the frequency is $1.27334 \times 10^{14} \mathrm{~Hz}$.
d. Write, explain and show the relevant formulas of pulse spread for the different types of dispersion that is effective in optical fibers.

Answer:
a. Dispersion is the phenomena that each frequency component will travel with different velocity due to the fact that the refractive index of the medium is a function of the wavelength.
b. Dispersion results in the broadening of the light pulse. This pulse broadening causes a pulse to overlap with the neighbouring pulses. After a certain amount of overlap has occurred, adjacent pulses can no longer be individually distinguished at the receiver and errors will occur. Thus, dispersion properties determine the limit of the information capacity of the fiber, i.e., the data bit rate limit.
c. $\quad v=\frac{\omega}{k}=\frac{2 \pi f}{2 \pi / \lambda}=f \lambda=1.27334 \times 10^{14} \times 1.55 \times 10^{-6}=1.9737 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
d. Material dispersion is due to the refractive index, $n$ being a function of the wavelength, $\lambda \Rightarrow \beta=\frac{2 \pi}{\lambda} n(\lambda)$. Important for single mode fibers and LED systems since LED has broader spectrum than laser diode. Group delay resulting from material dispersion is $t_{\text {mat }}=\frac{L}{c}\left(n-\lambda \frac{\mathrm{d} n}{\mathrm{~d} \lambda}\right)=\frac{L}{c} N_{g}$ where $N_{g}=n-\lambda \frac{\mathrm{d} n}{\mathrm{~d} \lambda}$ and the pulse spread due to material dispersion is $\tau_{\text {mat }}=\frac{\mathrm{d} t_{\text {mat }}}{\mathrm{d} \lambda} \sigma_{\lambda}=-\frac{L}{c} \sigma_{\lambda} \lambda \frac{\mathrm{d}^{2} n}{\mathrm{~d} \lambda^{2}}$

Waveguide dispersion is due to the explicit dependence of $\beta$ on $\omega$, i.e., $\beta=\frac{\omega}{c} n$, i.e., $\beta$ varies $\omega$ even though there is no material dispersion (i.e., $n \neq n(\omega)$ ). Pulse spread due to material dispersion
is $\tau_{w g}=-\frac{n_{2} L \Delta \sigma_{\lambda}}{c \lambda} V \frac{\mathrm{~d}^{2}(V b)}{\mathrm{d} V^{2}}$ where $V$ is the $V$ number of the fiber, $b=\frac{\frac{\beta}{k}-n_{2}}{n_{1}-n_{2}}$

Intermodal dispersion is due to the delays occurring between the different modes. In step index fiber, pulse spread due to waveguide dispersion is $\tau_{\bmod }=\frac{L}{c} N_{g 1}-\frac{L}{c} N_{g 2}$ where $N_{g}=n-\lambda \frac{\mathrm{d} n}{\mathrm{~d} \lambda}$.
For no material dispersion $N_{g}=n$, thus $\tau_{\bmod }=\frac{L}{c}\left(n_{1}-n_{2}\right)$.

Intermodal dispersion dominates pulse spreading in step index fibers.
27. a. In a fiber link, the connector loss is 0.5 dB for each connector. There are 10 splices in the link having loss of $0.1 \mathrm{~dB} /$ splice. The optical power at the end of the fiber flylead attached to the source is 1 mwatt . The receiver sensitivity is $-30 \mathrm{~dB}_{\mathrm{m}}$. In the link design 10 dB system margin is used. Find the fiber attenuation in $\mathrm{dB} / \mathrm{km}$ if the repeaterless link length is 50 km .
b. In the same link in part a, the laser diode, together with its drive circuitry has rise time of 3 ns , the spectral width of the laser diode is 1 nm , material dispersion factor is $0.1 \mathrm{~ns} /(\mathrm{nm} . \mathrm{km})$, bandwidth of the fiber is $2 \mathrm{GHz} . \mathrm{km}$, the power of the link length in the modal dispersion rise time is 0.7 and the 3 dB electric bandwidth of the receiver used is 100 MHz . Find the total rise time of this fiber link.
c. Is the design in part $b$ which is based on the rise time budget satisfactory if it is required that the fiber link should support a data bit rate of $1 \mathrm{~Gb} / \mathrm{s}$ using NRZ format?
d. Is the design in part $b$ which is based on the rise time budget satisfactory if it is required that the fiber link should support a data bit rate of $1 \mathrm{~Gb} / \mathrm{s}$ using RZ format?
e. What will you do to complete the design in this question?

Answer: a.
$P_{T}($ Total optical power loss $)=P_{s}($ Optical power at the end of the fiber flylead attached to the source $)$

$$
-P_{R}(\text { Receiver sensitivity })
$$

$P_{s}=1$ mwatt $=10 \log (1$ mwatt $) \mathrm{dB}=10 \log \left(\frac{1 \text { mwatt }}{1 \text { mwatt }}\right) \mathrm{dB}_{\mathrm{m}}=0 \mathrm{~dB}_{\mathrm{m}}$
$P_{T}=0 \mathrm{~dB}_{\mathrm{m}}-\left(-30 \mathrm{~dB}_{\mathrm{m}}\right)=30 \mathrm{~dB}$
$P_{T}=2 \ell_{c}+\alpha_{f} L+N \ell_{s}+S M$ where $\ell_{c}$ is the loss per connector, $\alpha_{f}$ is the fiber attenuation in $\mathrm{dB} / \mathrm{km}, L$ is the ink length in $\mathrm{km}, \ell_{s}$ is the loss per splice in $\mathrm{dB}, \mathrm{N}$ is the number of splicies in the link, $S M$ is the system margin in dB .

Thus
$P_{T}=2 \ell_{c}+\alpha_{f} L+N \ell_{s}+S M \Rightarrow 30 \mathrm{~dB}=2 \times 0.5 \mathrm{~dB}+\alpha_{f}(\mathrm{~dB} / \mathrm{km}) 50 \mathrm{~km}+10$ splices $\times 0.1 \mathrm{~dB} /$ splice +10 dB
$30 \mathrm{~dB}=1 \mathrm{~dB}+50 \alpha_{f} \mathrm{~dB}+1 \mathrm{~dB}+10 \mathrm{~dB}=\left(12+50 \alpha_{f}\right) \mathrm{dB} \Rightarrow \alpha_{f}=\frac{18}{50}=0.36 \mathrm{~dB} / \mathrm{km}$
b. $t_{s y s}=\left(t_{t x}^{2}+t_{m a t}^{2}+t_{\text {mod }}^{2}+t_{r x}^{2}\right)^{1 / 2}$ where $t_{s y s}$ is the total rise time of the link, $t_{t x}, t_{m a t}, t_{\text {mod }}$ and $t_{r x}$ are the rise times of the transmitter, material dispersion of the fiber, modal dispersion and the receiver, respectively. $t_{t x}$ is given as 3 ns ,
$t_{\text {mat }}=D_{\text {mat }} \sigma_{\lambda} L$ where $D_{\text {mat }}$ is the material dispersion factor in ns $/(\mathrm{nm} . \mathrm{km}), \sigma_{\lambda}$ is the spectral width of the laser diode so $t_{\text {mat }}=D_{\text {mat }} \sigma_{\lambda} L=0.1 \mathrm{~ns} /(\mathrm{nm} . \mathrm{km}) \times 1 \mathrm{~nm} \times 50 \mathrm{~km}=5 \mathrm{~ns}$,
$t_{\text {mod }}=\frac{440 L^{q}}{B_{0}}$ where $t_{\text {mod }}$ is in MHz, $L$ is the link length in $\mathrm{km}, q$ is given as $0.7, B_{0}$ is the bandwidth in 1 km of fiber in MHz so $t_{\mathrm{mod}}=\frac{440 L^{q}}{B_{0}}=\frac{440 \times 50^{0.7}}{2000} \approx 3.4 \mathrm{~ns}$
$t_{r x}$ results from the photodetector rise time and the3 dB electric bandwidth of the receiver front end ( $B_{r x}$ in MHz) and $t_{r x}=350 / B_{r x}$ ns. Thus $t_{r x}=350 / B_{r x}=350 / 100=3.5 \mathrm{~ns}$

So $t_{s y s}=\left(t_{t x}^{2}+t_{\text {mat }}^{2}+t_{\bmod }^{2}+t_{r x}^{2}\right)^{1 / 2}=\left[(3 \mathrm{~ns})^{2}+(5 \mathrm{~ns})^{2}+(3.4 \mathrm{~ns})^{2}+(3.5 \mathrm{~ns})^{2}\right]^{1 / 2} \approx 7.6 \mathrm{~ns}$
c. For NRZ, $\frac{0.7}{\text { Data bit rate }}=\frac{0.7}{1 \times 10^{9} \mathrm{bps}}=0.7 \mathrm{~ns} \Rightarrow t_{\text {sys }}=7.6 \mathrm{~ns}>0.7 \mathrm{~ns}$ so the design in part b which is based on the rise time budget is not satisfactory.
d. For RZ, $\frac{0.35}{\text { Data bit rate }}=\frac{0.35}{1 \times 10^{9} \mathrm{bps}}=0.35 \mathrm{~ns} \Rightarrow t_{s y s}=7.6 \mathrm{~ns}>0.35 \mathrm{~ns}$ so the design in part b which is based on the rise time budget is not satisfactory.
e. We will try other designs with new choices of link elements having different specifications to fulfill the requirements.

## HOMEWORK-7

28. In a fiber link, the loss in the fiber alone is foreseen to be 20 dB . The modern fiber used has the below fiber loss curve. Find the link length if a laser at $\lambda=1.55 \mu \mathrm{~m}, 1.3 \mu \mathrm{~m}, 0.9 \mu \mathrm{~m}, 0.8 \mu \mathrm{~m}$ is used.


Answer: At $\lambda=1.55 \mu \mathrm{~m}$, from the figure, optical loss can be found to be approximately $0.2 \mathrm{~dB} / \mathrm{km}$, so the link length is $20 \mathrm{~dB} /(0.2 \mathrm{~dB} / \mathrm{km})=100 \mathrm{~km}$.

At $\lambda=1.3 \mu \mathrm{~m}$, from the figure, optical loss can be found to be approximately $0.4 \mathrm{~dB} / \mathrm{km}$, so the link length is $20 \mathrm{~dB} /(0.4 \mathrm{~dB} / \mathrm{km})=50 \mathrm{~km}$.

At $\lambda=0.9 \mu \mathrm{~m}$, from the figure, optical loss can be found to be approximately $1.4 \mathrm{~dB} / \mathrm{km}$, so the link length is $20 \mathrm{~dB} /(1.4 \mathrm{~dB} / \mathrm{km})=14.3 \mathrm{~km}$.

At $\lambda=0.8 \mu \mathrm{~m}$, from the figure, optical loss can be found to be approximately $2.35 \mathrm{~dB} / \mathrm{km}$, so the link length is $20 \mathrm{~dB} /(2.35 \mathrm{~dB} / \mathrm{km})=8.51 \mathrm{~km}$.
29. a. For a single mode fiber if the pulse spread due to material dispersion is $5 \mathrm{ps} /(\mathrm{nm}-\mathrm{km})$ at $\lambda=1.3 \mu \mathrm{~m}$. The spectral width of the laser is 2 nm . What will be link length due to material dispersion if a data rate of 1 Gbps is required?
b. In a multimode step index fiber, the refractive index of the core and the cladding are 1.49 and 1.47 , respectively and the link length is 30 km . Find the pulse spread originating from intermodal dispersion and the data rate that can be used in this fiber.

Answer: a. $5 \mathrm{ps} /(\mathrm{nm}-\mathrm{km}) \times 2 \mathrm{~nm} \times L \mathrm{~km}=10 L \mathrm{ps}=10 L \times 10^{-12} \mathrm{sec}$

$$
\begin{aligned}
& \rightarrow 1 /\left(10 L \times 10^{-12}\right) \sec ^{-1}=1 \mathrm{Gbps}=1 \times 10^{9} \mathrm{sec}^{-1} \\
& \rightarrow 0.1 \times 10^{12} / \mathrm{L}=1 \times 10^{9} \rightarrow L=\frac{0.1 \times 10^{12}}{1 \times 10^{9}}=0.1 \times 10^{3}=100 \mathrm{~km}
\end{aligned}
$$

b. $\tau_{\bmod }=\frac{L\left(n_{1}-n_{2}\right)}{c}=\frac{L\left(n_{1}-n_{2}\right)}{c}=\frac{30 \times 10^{3}(1.49-1.47)}{3 \times 10^{8}}=10 \times 10^{-5}(0.02) \mathrm{sec}=2 \mu \mathrm{sec}$

Data bit rate $=1 /(2 \mu \mathrm{sec})=0.5 \mathrm{Mbps}$
30. a. Write the formula for the output optical intensity of a semiconductor laser diode in terms of the gain of the cavity and the absorption coefficient.
b. When the gain of the selected modes in the a semiconductor laser diode is sufficient to exceed the optical loss during one round trip in the cavity, lasing occurs. Write the intensity after the round trip in the presence of the two mirrors at each end of the cavity.
c. Write the formula for the gain at the threshold when the cavity acts as an oscillator, i.e., at the lasing threshold.
d. In a laser diode, the dimensions forming the junction area are $0.5 \mu \mathrm{~m}$ and $5 \mu \mathrm{~m}$, threshold current density is 180 billion $\mathrm{Amp} / \mathrm{m}^{2}$, diode injection current density is 240 billion $\mathrm{Amp} / \mathrm{m}^{2}$, resonator cavity length is $500 \mu \mathrm{~m}$, internal quantum efficiency is 0.7 , wavelength is $\lambda=1.55 \mu \mathrm{~m}$, absorption coefficient of the lasing medium is $15 \mathrm{~cm}^{-1}$, reflectivity of both mirrors are 0.75 , Plank's constant is $6.62607004 \times 10^{-34}$ joule.second and the electron charge is $1.60217662 \times 10^{-19} \mathrm{Amp} . \mathrm{sec}$. Find the output power of the laser.

Answer:
a. $I(z)=I(0) \exp \{[g(h f)-\alpha(h f)] z\}$
b. $I(2 L)=I(0) R_{1} R_{2} \exp \{2 L[g(h f)-\alpha(h f)]\}$
c. $\quad g_{t h}=(2 L)^{-1} \ln \left[\left(R_{1} R_{2}\right)^{-1}\right]+\alpha$
d.
$P_{0}=\frac{A\left(J-J_{t h}\right) h f\left[\frac{1}{2 L} \ln \left(\frac{1}{R_{1} R_{2}}\right)\right]}{q\left[\alpha+\frac{1}{2 L} \ln \left(\frac{1}{R_{1} R_{2}}\right)\right]} \eta_{i}$
$=\frac{\left(0.5 \times 10^{-6} \mathrm{~m} \times 0.5 \times 10^{-6} \mathrm{~m}\right)\left(240 \times 10^{9} \mathrm{~A} / \mathrm{m}^{2}-180 \times 10^{9} \mathrm{~A} / \mathrm{m}^{2}\right) h f\left[\frac{1}{2 L} \ln \left(\frac{1}{R_{1} R_{2}}\right)\right]}{q\left[\alpha+\frac{1}{2 L} \ln \left(\frac{1}{R_{1} R_{2}}\right)\right]} \eta_{i}$
For the given values, the numerator is

$$
\begin{aligned}
& \left(0.5 \times 10^{-6} \mathrm{~m} \times 5 \times 10^{-6} \mathrm{~m}\right)\left(240 \times 10^{9} \mathrm{Amp} / \mathrm{m}^{2}-180 \times 10^{9} \mathrm{Amp} / \mathrm{m}^{2}\right) 6.62607004 \times 10^{-34} \mathrm{Joule} . \mathrm{sec} \\
& \times\left[3 \times 10^{8} /\left(1.55 \times 10^{-6}\right) \mathrm{sec}^{-1}\right]\left[\frac{1}{2 \times 500 \times 10^{-6} \mathrm{~m}} \ln \left(\frac{1}{0.75 \times 0.75}\right)\right] 0.7 \\
& =\left(2.5 \times 10^{-12}\right)\left(60 \times 10^{9} \mathrm{Amp}\right) 6.62607004 \times 10^{-34} \mathrm{Joule} . \mathrm{sec} \\
& \times\left[3 \times 10^{14} /(1.55) \mathrm{sec}^{-1}\right]\left[\frac{10^{3}}{\mathrm{~m}} \ln \left(\frac{1}{0.75 \times 0.75}\right)\right] 0.7 \\
& =\left(450 \times 10^{-20}\right) \mathrm{Amp} \times 6.62607004 \text { Joule.sec. } \mathrm{sec}^{-1} \cdot \mathrm{~m}^{-1}[1 /(1.55)]\left[\ln \left(\frac{1}{0.75 \times 0.75}\right)\right] 0.7 \\
& =7.74779 \times 10^{-18} \text { Amp.Joule.m }{ }^{-1}
\end{aligned}
$$

For the given values, the denominator is
$q\left[\alpha+\frac{1}{2 L} \ln \left(\frac{1}{R_{1} R_{2}}\right)\right]=1.60217662 \times 10^{-19}$ Amp. $\sec \left[15 \mathrm{~cm}^{-1}+\frac{1}{2 \times 500 \times 10^{-6} \mathrm{~m}} \ln \left(\frac{1}{0.75 \times 0.75}\right)\right]$
$=q\left[\alpha+\frac{1}{2 L} \ln \left(\frac{1}{R_{1} R_{2}}\right)\right]=1.60217662 \times 10^{-19}$ Amp. $\sec \left[1500 \mathrm{~m}^{-1}+\frac{1}{10^{-3}} \ln \left(\frac{1}{0.75 \times 0.75}\right) \mathrm{m}^{-1}\right]$
$=1.60217662 \times 10^{-19}$ Amp.sec. $\mathrm{m}^{-1}\left[1500+\frac{1}{10^{-3}} \ln \left(\frac{1}{0.75 \times 0.75}\right)\right]=3.3251 \times 10^{-16}$ Amp.sec. $\mathrm{m}^{-1}$

So $P_{0}=\frac{7.74779 \times 10^{-18} \text { Amp.Joule. } \mathrm{m}^{-1}}{3.3251 \times 10^{-16} \text { Amp.sec. } \mathrm{m}^{-1}}=0.02301 \frac{\text { Joule }}{\mathrm{sec}}=0.02301 \mathrm{watt}=23.01 \mathrm{mwatt}$

## HOMEWORK-8

31. a. Sketch the intensity distributions of $\mathrm{TEM}_{00}, \mathrm{TEM}_{01}, \mathrm{TEM}_{11}$ and $\mathrm{TEM}_{32}$ transverse (spatial) modes of semiconductor lasers.
b. What is the frequency separation in GHz of axial modes in a semiconductor laser if the resonator length (or the mirror separation) is $500 \mu \mathrm{~m}$ and the refractive index is 1.5 ?
c. What happens in part b if the resonator length (or the mirror separation) is reduced to $250 \mu \mathrm{~m}$ ?
d. If the intensity of the laser transition line (gain curve) is from the wavelength 1549 nm to 1551 nm , the center being at 1550 nm , what should be the resonator length to have single mode laser diode operation?

Answer: a.


$\mathrm{TEM}_{00}$
TEM $_{01}$
TEM
11
TEM
32
b. $\Delta f=\frac{c}{2 L n}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{sec}}{2 \times 500 \times 10^{-6} \mathrm{~m} \times 1.5}=\frac{3 \times 10^{14}}{1000 \times 1.5} \mathrm{sec}^{-1}=2 \times 10^{11} \mathrm{sec}^{-1}=200 \mathrm{GHz}$
c. $\Delta f=\frac{c}{2 L n}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{sec}}{2 \times 250 \times 10^{-6} \mathrm{~m} \times 1.5}=\frac{3 \times 10^{14}}{500 \times 1.5} \mathrm{sec}^{-1}=4 \times 10^{11} \mathrm{sec}^{-1}=400 \mathrm{GHz}$
d. For 1549 nm wavelength, the frequency is

$$
f=\frac{c}{\lambda}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{sec}}{1549 \times 10^{-9} \mathrm{~m}}=1.93673 \times 10^{14} \mathrm{sec}^{-1}=1.93673 \times 10^{14} \mathrm{~Hz}
$$

For 1551 nm wavelength, the frequency is

$$
f=\frac{c}{\lambda}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{sec}}{1551 \times 10^{-9} \mathrm{~m}}=1.93424 \times 10^{14} \mathrm{sec}^{-1}=1.93424 \times 10^{14} \mathrm{~Hz}
$$

i.e., the frequency separation from 1549 nm to 1551 nm is

$$
1.93673 \times 10^{14} \mathrm{~Hz}-1.93424 \times 10^{14} \mathrm{~Hz}=2.49739 \times 10^{11} \mathrm{~Hz}
$$

i.e., frequency separation from 1550 nm to 1551 nm is
$2.49739 \times 10^{11} \mathrm{~Hz} / 2=1.2487 \times 10^{11} \mathrm{~Hz} \approx 124.87 \mathrm{GHz}$
Thus if $\Delta f=\frac{c}{2 L n}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{sec}}{2 L \times 10^{-6} \mathrm{~m} \times 1.5}=\frac{10^{14}}{L(\text { in } \mu \mathrm{m})} \sec ^{-1}=\frac{10^{14}}{L(\text { in } \mu \mathrm{m})} \mathrm{Hz}=\frac{10^{5}}{L(\text { in } \mu \mathrm{m})} \mathrm{GHz}$ is larger than 124.87 GHz , then the laser diode will operate in single mode.

$$
\text { So } \frac{10^{5}}{L(\text { in } \mu \mathrm{m})}>124.87 \Rightarrow L<\frac{10^{5}}{124.87} \mu \mathrm{~m} \Rightarrow L<800.83 \mu \mathrm{~m}
$$

32. Compare laser diode and LED performances in term of rms bandwidth, output power, power launched into the fiber and modulation bandwidth.

Answer:

|  | Laser <br> Diode | LED |
| :--- | :---: | :---: |
| rms Bandwidth | $2-4 \mathrm{~nm}$ | $15-60 \mathrm{~nm}$ |
| Output power | $1-10$ <br> mwatt | $1-10 \mathrm{mwatt}$ |
| Power launched <br> into the fiber | $0.5-5 \mathrm{mw}$ | $0.03-0.3$ <br> mw |
| Modulation <br> bandwidth | $>500 \mathrm{MHz}$ | $<200 \mathrm{MHz}$ |

33. a. What types of detectors are used in optical fiber communication systems?
b. Write 6 of the requirements needed for a good photodetector.
c. In a pin photodiode, the steady state average optical power is 0.1 mwatt, width of the depletion layer is $300 \mu \mathrm{~m}$, reflectivity of the entrance face (anti-reflection coating) of the photodiode is 0.2 , absorption coefficient of the semiconductor material is $20 \mathrm{~cm}^{-1}$ and the wavelength of operation is $1.55 \mu \mathrm{~m}$. Find the number of electron-hole pairs generated per second.
d. For the photodiode given in part c , find the average photocurrent generated by the steady state average optical power.
e. Find the quantum efficiency of the photodetector given in part c .
f. What is the responsivity of the photodetector in part c ?
g. If the avalanche photodiode (APD) has the same characteristics as in part c , additionally the average current gain is 80 , find the responsivity of the APD.

Answer: a. Semiconductor pin photodetector and avalanche photo diode (APD).
b. High sensitivity (should be able to detect as low power as possible before the noise factor starts to limit the performance). This sensitivity should be at the wavelength of operation of the source.

Speed (should be fast enough, i.e., should have large enough bandwidth to follow the data rate being used).

Minimum addition of noise to system.
Low cost and long life.
Compatible size to optical fiber dimensions.
Insensitive to temperature variations.
c.
number of electron-hole pairs generated

$$
\begin{aligned}
& =\frac{P_{0}\left(1-e^{-\alpha_{s} w}\right)\left(1-R_{f}\right)}{h f}=\frac{0.1 \times 10^{-3} \mathrm{watt}\left(1-e^{-20 \mathrm{~cm}^{-1} \times 300 \mu \mathrm{~m}}\right)(1-0.2)}{6.62607004 \times 10^{-34} \mathrm{Joule} . \sec \frac{3 \times 10^{8} \mathrm{msec}^{-1}}{1.55 \times 10^{-6} \mathrm{~m}}} \\
& =\frac{0.08 \times 10^{-3} \mathrm{watt}\left(1-e^{-2000 \mathrm{~m}^{-1} \times 300 \times 10^{-6} \mathrm{~m}}\right)}{6.62607004 \times 10^{-34} \mathrm{Joule} . \mathrm{sec} \frac{3 \times 10^{8} \mathrm{msec}^{-1}}{1.55 \times 10^{-6} \mathrm{~m}}}=\frac{0.08 \times 10^{-3} \mathrm{Joule}^{-\mathrm{sec}^{-1}\left(1-e^{-2000 \times 300 \times 10^{-6}}\right)}}{6.62607004 \times 10^{-34} \mathrm{Joule} \frac{3 \times 10^{8}}{1.55 \times 10^{-6}}} \\
& =\frac{0.08 \times 10^{-3} \mathrm{sec}^{-1}\left(1-e^{-2000 \times 300 \times 10^{-6}}\right)}{6.62607004 \times 10^{-34} \frac{3 \times 10^{8}}{1.55 \times 10^{-6}}}=2.81451 \times 10^{14} \mathrm{sec}^{-1}
\end{aligned}
$$

d. $I_{p}=q \times$ number of electron-hole pairs generated

$$
=1.60217662 \times 10^{-19} \text { Amp. } \sec \times 2.81451 \times 10^{14} \mathrm{sec}^{-1}
$$

$\approx 45.1 \mu \mathrm{Amp}$
e. $\eta=\left(1-e^{-\alpha_{s} w}\right)\left(1-R_{f}\right)=\left(1-e^{-20 \mathrm{~cm}^{-1} \times 300 \mu \mathrm{~m}}\right)(1-0.2)=\left(1-e^{-2000 \times 300 \times 10^{-6}}\right) 0.8=0.361$
f. $\mathfrak{R}=\frac{I_{p}}{P_{0}}=\frac{45.1 \mu \mathrm{Amp}}{0.1 \mathrm{mwatt}}=\frac{45.1 \times 10^{-6} \mathrm{Amp}}{0.1 \times 10^{-3} \mathrm{watt}}=\frac{451 \times 10^{-3} \mathrm{Amp}}{\text { watt }}=\frac{0.451 \mathrm{Amp}}{\text { watt }}$
g. $\mathfrak{R}_{A P D}=M \frac{I_{p}}{P_{0}}=80 \times \frac{0.451 \mathrm{Amp}}{\text { watt }}=80 \times \frac{0.451 \mathrm{Amp}}{\text { watt }}=36.08 \frac{\mathrm{Amp}}{\text { watt }}$

## HOMEWORK-9

34. a. Draw the block diagram of optical fiber transmitter.
b. Draw the block diagram of optical fiber receiver.

Answer: a.

b.

35. a. An analog video signal carrying the information of one television channel is to be transmitted in an optical fiber communication system. The maximum frequency in the video signal is 6 MHz . Find the data bit rate in Mbps required to send this television channel if a quantization having 1024 levels is used.
b. A movy sent in part a lasts 2 hours. Find the total number of bits that form this movy.
c. If BER in part b is $10^{-6}$, find the number of bits received in error throughout the whole movy.
d. If the optical fiber in part a can support 1 Tbps data bit rate, total how many television channels as defined in part a can be transmitted in the optical fiber communication link?

Answer: a. Nyquist rate requires no of samples of 2 x maximum frequency in one second

$$
=2 \times 6 \times 10^{6} \text { samples } / \mathrm{sec}=12 \times 10^{6} \text { samples } / \mathrm{sec} .
$$

Quantization having 1024 levels means each sample is presented by 10 bits since $1024=2^{10}$.
Data bit rate required to send this television channel $=12 \times 10^{6}$ samples $/ \mathrm{sec} \times 10$ bits $/$ sample

$$
=120 \mathrm{Mbps} .
$$

b. Total number of bits that form this movy $=120 \mathrm{Mbps} \times 60 \mathrm{sec} / \mathrm{min} \times 60 \mathrm{~min} /$ hour $\times 2$ hours $=864000 \times 10^{6}$ bits $=0.864$ Tbits.
c. Number of bits received in error throughout the whole movy $=0.864$ Tbits $\times 10^{-6}$ $=0.864 \times 10^{12}$ bits $\times 10^{-6}=0.864 \times 10^{6}$ bits.
d. Total number of television channels that can be transmitted is $1 \mathrm{Tbps} / 120 \mathrm{Mbps}$ $=8333$ channels.
36. In a fiber transmission, bit duration is 1 ns .
a. Find the data bit rate.
b. If the system rise time is 0.6 ns, would you prefer to use NRZ (non-return-to-zero) or RZ (return-to-zero) encoding?

Answer: a. Data bit rate is $1 / 1 \mathrm{~ns}=1 \mathrm{Gbps}$.
b. For NRZ, $t_{\mathrm{s}} \leq 0.7 / B_{\mathrm{r}}$ in order to have acceptable reception where $B_{\mathrm{r}}$ is the data bit rate. $0.7 / B_{\mathrm{r}}=0.7 / 1 \mathrm{Gbps}=0.7 \mathrm{~ns}$, i.e., $0.6 \mathrm{~ns} \leq 0.7 \mathrm{~ns}$ fulfills the acceptable criterion to avoid distortion.

For RZ, $t_{\mathrm{s}} \leq 0.35 / B_{\mathrm{r}}$ in order to have acceptable reception.
$0.35 / B_{\mathrm{r}}=0.35 / 1 \mathrm{Gbps}=0.35 \mathrm{~ns}$ which doesnot satisfy $t_{\mathrm{s}} \leq 0.35 / B_{\mathrm{r}}$ so the acceptable criterion to avoid distortion is not fulfilled so RZ can not be used.
37. Data sequence is 0110011101 and the clock is

a. Draw the NRZ coded signal.
b. Draw the RZ coded signal.
c. Draw the Manchester coded signal.

Answer: a.

b.

c.

38. In an optical fiber receiver, what needs to be done to overcome the noise limitations and achieve an
acceptable reception.
Answer: Minimum signal to noise ratio (SNR) is preset for an acceptable system performance, Minimum average detector current is determined to satisfy the SNR requirement, Minimum detectable optical power is found. Includes the choice of pin or APD photodetectors.
39. Write the types of modulation in optical fiber communication.

Answer: Analog modulation covers the direct modulation of optical carrier power by the baseband signal and modulation of optical carrier power by a frequency modulated subcarrier.

Digital modulation covers the intensity modulation.
40. What are the sources of receiver noise?

Answer: Photodetector noises such as quantum noise and dark noise (photodetector noise in the absence of light,

Thermal noise (thermal motion of charge carriers) due to the amplifier input resistance, the bias (load) resistance and the photodetector resistance,

Noise originating from the active electronic device such as a transistor. The magnitude of this noise depends on the material ( $\mathrm{Si}, \mathrm{Ge}, \ldots$ etc) and design of the device and its bias.

## HOMEWORK-10

41. Write 5 categories that WOC (Wireless Optical Communication) can be classifed based on their transmission range.

Answer:
(i) Ultrashort-range WOC - used in chip-to-chip communication or all optical lab-on-a-chip system.
(ii) Short-rangeWOC - used in wireless body area networks (WBANs) or wireless personal area networks (WPANs).
(iii) Medium-range WOC - used in indoor IR or visible light communication (VLC) for wireless local area networks (WLANs) and inter-vehicular and vehicle-to-infrastructure communications.
(iv) Long-range WOC - used in terrestrial communication between two buildings or metro area extensions.
(v) Ultra-long-range WOC - used in ground-to-satellite/satellite-to-ground or inter-satellite link or deep space missions.
42. Draw the block diagram of WOC.

Answer:


24
43. a. In a 3 km FSO link operating at 1300 nm wavelength, $\beta_{T(\text { overall })}=0.5 \mathrm{~km}^{-1}$. Visibility is 7 km and the size distribution of the scattering rain particles is 1.6 .
a. Find the scattering coefficient of rain.
b. What is the overall transmittance?
c. What is the overall transmittance found in part a composed of?

Answer:
a. $\quad \beta_{\text {rain }(\text { satat })}=\left(\frac{3.91}{V}\right)\left(\frac{0.55}{\lambda}\right)^{i}=\left(\frac{3.91}{7}\right)\left(\frac{0.55}{1.3}\right)^{1.6}=0.14 \mathrm{~km}^{-1}$
b. Overall transmittance is $\tau_{\text {overall }}=\exp \left(-\beta_{T(\text { overall })} L\right)=\exp \left(-0.5 \mathrm{~km}^{-1} \times 3 \mathrm{~km}\right)=\exp (-1.5)=0.22$
c. Overall transmittance is composed of the transmittances originating from aerosol scattering, aerosol absorption, molecular scattering, molecular absorption and turbulence scattering, i.e.,

$$
\tau_{\text {overall }}=\exp \left(-\beta_{T(\text { overall })} L\right)=\exp \left[-\left(\beta_{a(\text { scat })}+\beta_{a(\text { absorp })}+\beta_{m(\text { scat })}+\beta_{m(\text { absorp })}+\beta_{t(\text { scat })}\right) L\right] .
$$

44. For the following transmittance curve, write 3 wavelengths that will properly operate an FSO link and 3 wavelengths that will not allow the operation of the FSO link.


Answer: $1300 \mathrm{~nm}, 1550 \mathrm{~nm}, 2250 \mathrm{~nm}$ will properly operate an FSO link, $1400 \mathrm{~nm}, 1900 \mathrm{~nm}, 1150 \mathrm{~nm}$ will not allow the operation of the FSO link.

## HOMEWORK-11

45. In an FSO system operating at 900 nm , the laser source is a Gaussian-beam wave at the source plane $(z=0)$. The amplitude of the electric field at the origin is $1 \mathrm{mV} / \mathrm{m}$, effective Gaussian beam radius (spot size) is 5 cm , the link length and the focal length is 500 m .
a. Write the complex amplitude of the Gaussian-beam wave at the source plane $(z=0)$.
b. Find the intensity at the source plane.
c. Find the electric field at the receiver plane if the link length is 500 m .
d. Find the intensity at the receiver plane for part c .
e. What is the value of the intensity at the origin of the receiver plane in part d.

Answer: a.

$$
\begin{aligned}
U_{0}(\mathbf{s}, 0) & =A \exp \left(-\frac{1}{2} \alpha_{0} k s^{2}\right)=A \exp \left[\frac{i k}{2 z}\left(i \alpha_{0} z\right) s^{2}\right]=A \exp \left(-\frac{s^{2}}{W_{0}^{2}}-\frac{i k}{2 F_{0}} s^{2}\right) \\
& =1 \times 10^{-3}(\mathrm{~V} / \mathrm{m}) \exp \left[-\frac{s^{2}}{\left(5 \times 10^{-2}\right)^{2}}-\frac{i 2 \pi /\left(900 \times 10^{-9}\right)}{2 \times 500} s^{2}\right]=\exp \left(-400 s^{2}-\frac{i 2 \pi}{9 \times 10^{-4}} s^{2}\right)(\mathrm{mV} / \mathrm{m}) \\
& =\exp \left(-400 s^{2}-i 6981.32 s^{2}\right)(\mathrm{mV} / \mathrm{m})
\end{aligned}
$$

b.

$$
\begin{aligned}
I(\mathbf{s}, 0) & =U_{0}(\mathbf{s}, 0) U_{0}^{*}(\mathbf{s}, 0)=\exp \left(-400 s^{2}-i 6981.32 s^{2}\right) \exp \left(-400 s^{2}+i 6981.32 s^{2}\right) \mu \mathrm{watt} / \mathrm{m}^{2} \\
& \approx \exp \left(-800 s^{2}\right) \mu \mathrm{watt} / \mathrm{m}^{2}
\end{aligned}
$$

c.

$$
U(\mathbf{r}, z)=\frac{A}{1+i \alpha_{0} z} \exp \left[i k z+\frac{i k}{2 z}\left(\frac{i \alpha_{0} z}{1+i \alpha_{0} z}\right) r^{2}\right]
$$

$$
=\frac{1 \times 10^{-3}}{1+i\left(\frac{2}{k W_{0}^{2}}+i \frac{1}{F_{0}}\right) z} \exp \left\{i k z+\frac{i k}{2 z}\left[\frac{i\left(\frac{2}{k W_{0}^{2}}+i \frac{1}{F_{0}}\right) z}{1+i\left(\frac{2}{k W_{0}^{2}}+i \frac{1}{F_{0}}\right) z}\right] r^{2}\right\}
$$

$$
=\frac{1 \times 10^{-3}}{1+i\left\{\frac{2\left(900 \times 10^{-9}\right)}{2 \pi\left(5 \times 10^{-2}\right)^{2}}+i \frac{1}{500}\right\} 500}
$$

$$
\times \exp \left\{i\left[2 \pi /\left(900 \times 10^{-9}\right)\right] 500+\frac{i\left[2 \pi /\left(900 \times 10^{-9}\right)\right]}{2 \times 500}\left[\frac{i\left(\frac{2}{\left[2 \pi /\left(900 \times 10^{-9}\right)\right]\left(5 \times 10^{-2}\right)^{2}}+i \frac{1}{500}\right) 500}{1+i\left(\frac{2}{\left[2 \pi /\left(900 \times 10^{-9}\right)\right]\left(5 \times 10^{-2}\right)^{2}}+i \frac{1}{500}\right) 500}\right] r^{2}\right\}
$$

$$
=-i 17.54 \exp \left[i 0.35 \times 10^{10}+(i 69984.6-122780) r^{2}\right](\mathrm{mV} / \mathrm{m})
$$

d.

$$
\begin{aligned}
I(\mathbf{r}, z)=U(\mathbf{r}, z) U^{*}(\mathbf{r}, z)= & -i 17.54 \exp \left[i 0.35 \times 10^{10}+(i 69984.6-122780) r^{2}\right] \\
& \times i 17.54 \exp \left[-i 0.35 \times 10^{10}+(-i 69984.6-122780) r^{2}\right] \\
& =307.7 \exp \left(-245560 r^{2}\right)\left(\mu \mathrm{watt} / \mathrm{m}^{2}\right)
\end{aligned}
$$

e. The value of the intensity at the origin of the receiver plane in part d is the intensity found in part d evaluated at $r=0$ which is $307.7\left(\mu \mathrm{watt} / \mathrm{m}^{2}\right)$
46. In an FSO link, find the optical power at the receiver if transmitted optical power at the transmit interface is 1 mwatt, transmit optics efficiency is 0.9 , aperture illumination efficiency of the transmitter lens is 0.8 , wavelength is 900 nm , transmitting aperture area is $3 \mathrm{~cm}^{2}$, fractional transmitter pointing loss is 0.8 , fractional loss due to scattering and absorption of the atmosphere is 0.6 , fractional signal loss due to mismatch of the transmitting and receiving polarization patterns is 0.95 , fractional receiver pointing loss is 0.9 , receiving aperture area is $6 \mathrm{~cm}^{2}$, link distance is 1 km and receiving optics collecting efficiency is 0.8 .

Answer:

$$
P_{R}=P_{T}\left(\eta_{T} \eta_{A} \frac{4 \pi A_{T}}{\lambda^{2}}\right) L_{T P} L_{a t m} L_{p o l} L_{R P}\left(\frac{A_{R}}{4 \pi z^{2}}\right) \eta_{R}
$$

where $P_{R}$ is the total signal power at the input to the receiver, $P_{T}$ is the transmit optical power at the transmit interface, $\eta_{T}$ is the transmitter optics efficiency, $\eta_{A}$ is the aperture illumination efficiency of the transmitter lens, $\lambda$ is the wavelength, $A_{T}$ is the transmitter aperture area, $L_{T P}$ is the fractional transmitter pointing loss, $L_{\text {atm }}$ is the fractional loss due to absorption of the atmosphere, $L_{p o l}$ is the fractional signal loss due to mismatch of the transmitting and receiving polarization patterns, $L_{R P}$ is the fractional receiver pointing loss, $A_{R}$ is the receiver aperture area, $z$ is the link distance and $\eta_{R}$ is the receiving optics collecting efficiency. Thus,
$P_{R}=1\left[0.9 \times 0.8 \frac{4 \pi 3 \times 10^{-4} \mathrm{~m}^{2}}{\left(900 \times 10^{-9}\right)^{2}}\right] 0.8 \times 0.6 \times 0.95 \times 0.9\left[\frac{6 \times 10^{-4} \mathrm{~m}^{2}}{4 \pi\left(1 \times 10^{3}\right)^{2} \mathrm{~m}^{2}}\right] 0.8(\mathrm{mwatt})=0.0525(\mathrm{mwatt})$
47. a. Write the signal-to-noise ratio (SNR) formula when pin photodiode is used in an FSO link.
b. Write the signal-to-noise ratio (SNR) formula when avalnche photodiode (APD) is used in an FSO link.

Answer: a

$$
S N R=\frac{\left(R_{0} P_{R}\right)^{2}}{2 q B\left(R_{0} P_{R}+R_{0} P_{B}+I_{d}\right)+4 K_{B} T B / R_{L}}
$$

where $B$ is the receiver bandwidth, $I_{d}$ is the dark current, $K_{B}=1.3807 \times 10^{-23}$ joules per kelvin $\left(J \cdot K^{-1}\right)$ is the Boltzmann's constant, $T$ is the absolute temperature, $R_{L}$ is the equivalent load resistance, $P_{B}$ is the background noise power and $R_{0}$ in $\mathrm{mA} /$ watt is the detector responsivity given by
$R_{0}=\frac{\eta q}{h f}$
where $\eta$ is the detector quantum efficiency, $q=1.602 \times 10^{-19}$ Coulomb is the electronic charge, $h=6.623 \times 10^{-34}$ Joule.sec is the Planck's constant, $f$ the operating frequency.
b. When APD is used, the dark current and shot noise are increased by the multiplication process; however, the thermal noise remains unaffected. Therefore, if the photocurrent is increased by a factor of $M$ avalanche multiplication factor, then the total shot noise is also increased by the same factor. The direct detection SNR for APD photodetector is
$S N R=\frac{\left(M R_{0} P_{R}\right)^{2}}{\left[2 q B\left(R_{0} P_{R}+R_{0} P_{B}+I_{d b}\right) M^{2} F+I_{d s}\right]+4 K_{B} T B / R_{L}}$
where $F$ is the excess noise factor arising due to random nature of multiplication factor, $I_{d b}$ is the bulk dark current, and $I_{d s}$ is the surface dark current.

## HOMEWORK-12

48. a. Write the wavelengths used in optical wireless communication systems operating in various underwater media.
b. What are the causes of total attenuation of optical waves in underwater medium?
c. What is the extinction coefficient in underwater medium and what is it composed of?
d. What are some of the constituents in underwater medium that cause the absorption and scattering of optical beam?
e. What are the four major water types that change the propagation behaviour of optical waves while they propagate in an underwater medium?

Answer: a. Seawater shows a decreased absorption in the blue/green region of the visible spectrum. Thus, using suitable wavelengths, for instance in the blue/green region, high speed connections can be attained according to the type of water ( $400-500 \mathrm{~nm}$ for clear to $300-700 \mathrm{~nm}$ for turbid water conditions).
Minimum attenuation is centered near $0.460 \mu \mathrm{~m}$ in clear waters and shifts to higher values for dirty waters approaching $0.540 \mu \mathrm{~m}$ for coastal waters.
b. Causes of total attenuation of optical waves in underwater medium are the absorption and scattering due to various constituents existing in water.
c. $c(\lambda)$ in $m^{-1}$ is the extinction coefficient expressing the total attenuation occurred by the propagation through the water. The total attenuation is composed of the absorption and scattering. Thus,
$c(\lambda)=\alpha(\lambda)+\beta(\lambda)$ where $\alpha(\lambda)$ is the absorption coefficient, $\beta(\lambda)$ is the scattering coefficient.
d. Underwater medium contains almost 80 different elements, dissolved or suspended in pure water, with different concentrations. Some of them are

- Various dissolved salts such as $\mathrm{NaCl}, \mathrm{MgCl} 2$, etc, which absorb light at specific wavelengths and induce scattering effects.
- Minerals like sand, metal oxides, which contribute to both absorption and scattering.
- Colored dissolved organic matters such as fulvic and humic acids which affect absorption, mainly in blue and ultraviolet wavelengths.
- Organic matters such as viruses, bacteria, and organic detritus which add backscattering, especially in the blue spectral range.
- Phytoplankton with chlorophyll-A which strongly absorbs in the blue-red region and scatters green light.
e. The four major water types are
- Pure deep ocean waters cobalt blue where the absorption is high and the scattering coefficient is low.
- Clear sea waters with higher scattering due to many dissolved particles.
- Near coasts ocean waters with absorption and scattering due to planktonic matters, detritus and mineral components.
- Harbor murky waters, which are quite constraining for optical propagation due to dissolved and insuspension matters.

49. Find the received power in underwater medium according to Beer's Law if the transmitted power is 1 mwatt, the link length is 50 meters, total absorption coefficient due to all the constituents in water is $0.01 \mathrm{~m}^{-1}$ and the total scattering coefficient due to all the constituents in water is $0.005 \mathrm{~m}^{-1}$.

Answer: $P(z)=P_{0} e^{-c(\lambda) z}=1 e^{-\left(0.01 \mathrm{~m}^{-1}+0.005 \mathrm{~m}^{-1}\right) 50 \mathrm{~m}} \mathrm{mwatt}=e^{-(0.015) 50} \mathrm{mwatt}=e^{-0.75} \mathrm{mwatt}=0.472 \mathrm{mwatt}$
50. What is oceanic turbulence, what causes it and what is its effect?

Answer: Optical wireless communications are greatly affected by optical turbulence, which refers to random fluctuations of the refraction index.

In the case of underwater systems, these fluctuations are mainly caused by variations in temperature and salinity of the oceanic water.

An important parameter for the description of oceanic turbulence is the scintillation index, which expresses the variance of the wave intensity.
51. In a link budget calculation of an underwater optical wireless communication link, find the received optical power under LOS conditions if the transmitted power is 2 mwatts, optical efficiencies of the transmitter and the receiver are 0.8 each, total extinction coefficient is $0.001 \mathrm{~m}^{-1}$, perpendicular distance between the transmitter plane and receiver plane 10 m , transmittance beam divergence angle is $5^{0}$, angle between the perpendicular to the receiver plane and the transmitter-receiver trajectory is $0^{0}$
and the receiver aperture area is $100 \mathrm{~cm}^{2}$.
Answer:

$$
\begin{aligned}
P_{R} & =P_{T} \eta_{t} \eta_{r} e^{-\frac{c(\lambda) R}{\cos \theta}} \frac{A_{R} \cos (\theta)}{2 \pi R^{2}\left(1-\cos \theta_{0}\right)}=2 \times 0.8 \times 0.8 e^{-\frac{0.001 \mathrm{~m}^{-1} 10 \mathrm{~m}}{\cos \left(0^{0}\right)}} \frac{100 \mathrm{~cm}^{2} \times \cos \left(0^{0}\right)}{2 \pi \times 10^{2} \mathrm{~m}^{2}\left[1-\cos \left(5^{0}\right)\right]}(\text { mwatt }) \\
& =1.28 e^{-0.01} \frac{10^{-2} \mathrm{~m}^{2}}{2 \pi \times 10^{2} \mathrm{~m}^{2}\left[1-\cos \left(5^{0}\right)\right]}(\text { mwatt })=0.54 \text { mwatt }
\end{aligned}
$$

52. What is aimed in all optical networks?

Answer: The aim in all optical networks is to get rid of the data bit rate limitation imposed by electronics in telecommunication networks and use total optics in all the devices in telecommunication networks so that extremely high data bit rate is achieved at each point of the telecommunication network.
53. a. What are currently employed optical elements in the telecommunications network?
b. What are the optical elements that are not yet currently employed in the telecommunications network?

Answers: a.

- Optical fibers:

First, more capacity between two sites meant the installation of more fibers.
Then more time division multiplexed (TDM) signals are placed in the same fiber, i.e. the bandwith handling capability of the fibers were increased. (both through fiber manufacturing and semiconductor laser modulation techniques supporting high rates of 40 Gbps .)

- Optical networks with Dense Wavelength Division Multiplexing (DWDM) provide additional capacity on existing fibers. DWDM is introduced providing many virtual fibers on a single physical fiber which increased drastically the information rate carrying capability of fibers (in the order of hundreds of Terabits per second).

- SDH/SONET.
- Optical Amplifiers
- Erbium-Doped Fiber Amplifier (EDFA). By doping a small strand of fiber with a rare earth metal, such as erbium, optical signals could be amplified without converting the signal back to an electrical state.
- EDFA operating at 1550 nm is used at each $50-100 \mathrm{~km}$ and replaces electronic regenerators.
- EDFA enables data rates of 10 Gbps or higher. With the electronic conversion the rate was limited by 2.5 Gbps .
- Laser diodes used in optical fiber communications.
- LED light sources.
- Optical detectors used in optical fiber communications.
- Tunable Lasers:

Radiate light at different wavelengths.
Can switch from one wavelength to another very quickly.

- Narrowband Lasers

Advanced lasers have extremely narrow source spectral bandwidths ( $\ll 1 \mathrm{~nm}$ ), very narrow wavelength spacings.

Long-haul applications use externally modulated lasers, while shorter applications can use integrated laser technologies.
b. Optical elements that are not yet currently employed in the telecommunications network:

- Optical Switches (Sometimes referred to as Optical Cross Connects or Wavelength Routers)

Switch takes traffic in electrical form from an input port or connection and directs it again in electrical form over a backplane, to an output port.

Electronic switches direct variable-length packets, fixed-length cells, and synchronous timeslots from an input port to an output port.

An optical switch works with light. It directs a light beam of a single wavelength or of a range of wavelengths from an input port to an output port.

- Optical Add/Drop Multiplexers

Fiber Bragg Gratings

- It is a small section of fiber modified to create periodic changes in the index of refraction.
- Depending on the space between the changes, a certain frequency of light - the Bragg resonance wavelength - is reflected back, while all other wavelengths pass through.

- Optical filters: Fiber Bragg gratings are also used in signal filtering.
- Multiplexers, demultiplexers


## Thin Film Substrates

- By coating a thin glass or polymer substrate with a thin interference film of dielectric material, the substrate can be made to pass through only a specific wavelength and reflect all others.
- By integrating several of these components, optical network devices such as multiplexers, demultiplexers and add/drop devices are designed.

54. What are the types of optical switches:

Answer:

- MEMS (Micro Electro Mechanical System) Switches:

Light in one fiber is just redirected to move to a different fiber by using microscopic (with diameters of a human hair) moveable (moveable in three dimensions) mirrors (several hundred mirrors placed together on mirror arrays in an area of a few centimeters square).

Light from an input fiber is aimed at a mirror, which is directed to move the light to another mirror on a facing array.

Light beams themselves tell the mirror (through digital wrappers) what bend to make in order to route the light appropriately.

This mirror then reflects the light down towards the desired output optical fiber.
There exists designs of $1,024 \times 1,024$ wavelengths (if each can carry 40 Gbps it corresponds to a capacity of $40 \mathrm{Gbps} \times 1,024=40.96 \mathrm{Tbps}$ ) in an area of around 25 cm x 15 cm .

Picture of a MEMS mirror and MEMS mirror array deflection mechanism are shown below:


MEMS Mirror


- Buble Switches: Use heat to create small bubles in fluid channels which then reflect and direct light
- Thermo-optical Switches:

Light passing through glass is heated up or cooled down by using electrical coils.
Heat alters the refractive index of the glass, bending the light to enter one fiber or another.

- Liquid Crystal (LCD) Switches:

Use liquid to bend light


Liquid Crystal Cell

Wavelength Switching:
Single wavelength enters the switch
A "wavelength" selection is made by using prisms, filters or gratings.
Based on the wavelength selected, the light is switched to a known output port.

- Optical Burst Switching:

Disadvantage of lambda switching is that, once a wavelength has been assigned, it is used exclusively by its "owner."

If 100 percent of its capacity is not in use for 100 percent of the time, then there is an inefficiency in the network.

A solution to this is to allocate the wavelength for the duration of the data burst being sent giving rise to optical burst switching.

- Optical Packet Switching (OPS):

OPS is the optical equivalent of an electronic packet switch, reading the embedded label and making a switching decision using this information.

- Holographic Switching:
- Creates a wavelength-specific reflective grating, but does this dynamically.
- The grating structure in these devices is written as a hologram into a piece of glass.
- The holograms are "invisible" until they are energized by a set of control electrodes.


